# Mathematics

# Natural Sciences

# Requirements

For well over two thousand years, mathematics has been a part of the human search for understanding. Mathematical discoveries have come both from the attempt to describe the natural world and from the desire to arrive at a form of inescapable truth through careful reasoning that begins with a small set of self-evident assumptions. These remain fruitful and important motivations for mathematical thinking, but in the last century mathematics has been successfully applied to many other aspects of the human world: voting trends in politics, the dating of ancient artifacts, the analysis of automobile traffic patterns, and long-term strategies for the sustainable harvest of deciduous forests, to mention a few. Today, mathematics as a mode of thought and expression is more valuable than ever before. Learning to think in mathematical terms is an essential part of becoming a liberally educated person.

Mathematics is an engaging field, rich in beauty, with powerful applications to other subjects. Thus we strive to ensure that Kenyon students encounter and learn to solve problems using a number of contrasting but complementary mathematical perspectives: continuous and discrete, algebraic and geometric, deterministic and stochastic, theoretical and applied. In our courses we stress mathematical thinking and communication skills. And in courses where it makes sense to incorporate technological tools, our students learn to solve mathematical problems using computer algebra systems, statistical packages, and computer programming languages.

#### **New Students**

For those students who want only an introduction to mathematics, or perhaps a course to satisfy a distribution requirement, selection from MATH 105, 106, 111, 116, 128 and SCMP 118 is appropriate. Students who think they might want to continue the study of mathematics beyond one year, either by pursuing a major or minor in mathematics or as a foundation for courses in other disciplines, usually begin with the calculus sequence (MATH 111, 112, and 213). Students who have already had calculus or who want to take more than one math course may choose to begin with MATH 106 and MATH 206 or Introduction to Programming (SCMP 118). A few especially well-prepared students take MATH 224 or MATH 222 in their first year. Please see the department chair for further information.

MATH 111 is an introductory course in calculus. Students who have completed a substantial course in calculus might qualify for one of the successor courses, MATH 112 or 213. MATH 106 is an introduction to statistics, which focuses on quantitative reasoning skills and the analysis of data. SCMP 118 introduces students to computer programming.

To facilitate proper placement of students in calculus courses, the department offers placement tests that help students decide which level of calculus course is appropriate for them. This and other entrance

information is used during the orientation period to give students advice about course selection in mathematics. We encourage all students who do not have Advanced Placement credit to take the placement exam that is appropriate for them. Students who have Advanced Placement credit for MATH 106 should consider enrolling in MATH 206 or MATH 216.

The ready availability of powerful computers has made the computer one of the primary tools of the mathematician. Students will be expected to use appropriate computer software in many of the mathematics courses. However, no prior experience with the software packages or programming is expected, except in advanced courses that presuppose earlier courses in which use of the software or programming was taught.

# Course Requirements for the Major

There are two concentrations within the mathematics major: classical mathematics and statistics. The coursework required for completion of the major in each concentration is given below.

#### **Classical Mathematics**

A student must have credit for the following core courses:

- Three semesters of calculus (MATH 111, 112, 213, or the equivalent)
- One semester of statistics (MATH 106 or 436, or the equivalent)
- SCMP 118-Introduction to Programming
- MATH 222-Foundations
- MATH 224-Linear Algebra I
- MATH 335-Abstract Algebra I or MATH 341 Real Analysis I

In addition, majors must have credit for at least **three other elective** courses selected with the consent of the department. MATH 110 may not be used to satisfy the requirements for the major.

# **Statistics**

A student must have credit for the following core courses:

- Three semesters of calculus (MATH 111, 112, 213 or the equivalent)
- SCMP 118-Introduction to Programming
- MATH 222-Foundations
- MATH 224-Linear Algebra I
- MATH 336-Probability
- MATH 341-Real Analysis I

MATH 416-Linear Regression Models or MATH 436 Mathematical Statistics

In addition to the core courses, majors must also have credit for **two elective courses** from the following list:

- MATH 106-Elements of Statistics
- MATH 206-Data Analysis
- MATH 216-Nonparametric Statistics
- MATH 236-Random Structures
- MATH 416-Linear Regression Models
- MATH 436-Mathematical Statistics

# **Applications of Math Requirement**

Mathematics is a vital component in the methods used by other disciplines, and the applied math requirement is designed to expose majors to this vitality. There are two ways to satisfy the requirement:

- a) Earn credit for two courses (at least 1 unit) from a single department or program that use mathematics in significant ways. Typically, majors will choose a two-course sequence from the following list; other two-course sequences require departmental approval:
  - PHYS 140/145
  - ECON 101/102
  - PSYC 200 together with a 400-level Research Methods in Psychology course
- b) Earn credit for a single math course that focuses on the development and analysis of mathematical models used to answer questions arising in other fields. The following courses satisfy the requirement, but other courses may satisfy the requirement with approval of the department:
  - MATH 258-Mathematical Biology
  - MATH 347-Mathematical Models

Classical mathematics majors may also use MATH 206, MATH 216, MATH 226, or MATH 416 to satisfy the requirement. Additionally, students choosing this option may not use the applied math course as one of the elective courses required for the major.

# **Depth Requirement**

Majors are expected to attain a depth of study within mathematics, as well as breadth. Therefore majors should earn credit in one of four two-course upper-level sequences:

MATH 335/435-Abstract Algebra I & II

- MATH 341/441-Real Analysis I & II
- MATH 336/436-Probability and Mathematical Statistics
- MATH 336/416-Probability and Linear Regression Models

Other two-course sequences may satisfy the requirement with approval from the department.

# **Senior Exercise**

The Senior Exercise begins promptly in the fall of the senior year with independent study on a topic of interest to the student and approved by the department. The independent study culminates in the writing of a paper, which is due in November. (Juniors are encouraged to begin thinking about possible topics before they leave for the summer.) Students are also required to take the Major Field Test in Mathematics produced by the Educational Testing Service. Evaluation of the Senior Exercise is based on the student's performance on the paper and the standardized exam. A detailed guide on the Senior Exercise is available on the Math Department Web site under the link The Senior Exercise.

# **Suggestions for Majoring in Mathematics**

Students wishing to keep open the option of a major in mathematics typically begin with the study of calculus and normally complete the calculus sequence, MATH 222 (Foundations), and either SCMP 118 or MATH 106 by the end of the sophomore year. A major is usually declared no later than the second semester of the sophomore year. Those considering a mathematics major should consult with a member of the mathematics department to plan their course of study.

The requirements for the major are minimal. Anyone who is planning a career in the mathematical sciences, or who intends to read for honors, is encouraged to consult with one or more members of the department concerning further studies that would be appropriate. Similarly, any student who wishes to propose a variation of the major program is encouraged to discuss the plan with a member of the department prior to submitting a written proposal for a decision by the department.

Students who are interested in teaching mathematics at the high-school level should take MATH 230 (Geometry) and MATH 335 (Abstract Algebra I), since these courses are required for certification in most states, including Ohio.

# **Honors in Mathematics**

# Eligibility

To be eligible to enroll in the Mathematics Honors Seminar, by the end of junior year students must have completed one depth sequence (MATH 335-435, MATH 336-416, MATH 336-436, MATH 341-441) and have earned a GPA of at least 3.33, with a GPA in Kenyon mathematics courses of at least 3.6. The student must also have, in the estimation of the mathematics faculty, a reasonable expectation of fulfilling the requirements for Honors, listed below.

To earn Honors in mathematics, a student must: (1) Complete two depth sequences (see list above); (2) Complete at least six 0.5-unit courses in mathematics numbered 300 or above; (3) Pass the Senior Exercise in the fall semester; (4) Pass the Mathematics Honors Seminar MATH 498; (5) Present the results of independent work in MATH 498 to a committee consisting of an outside examiner and members of the Kenyon Mathematics Department; (6) successfully complete an examination written by an outside examiner covering material from MATH 498 and previous mathematics courses; (7) Maintain an overall Kenyon GPA of at least 3.33; (8) Maintain a Mathematics Department GPA of at least 3.6.

# **Awarding Honors**

Based on performance in all of the above-mentioned areas, the department (in consultation with the outside examiner) can elect to award Honors, High Honors, or Highest Honors; or not to award honors at all.

# Requirements for the Minors

There are two minors in mathematics. Each minor deals with core material of a part of the discipline, and each reflects the logically structured nature of mathematics through a pattern of prerequisites. A minor consists of satisfactory completion of the courses indicated.

#### **Mathematics**

The calculus sequence MATH 111, 112, 213, and four courses from the following: MATH 105, 106, 116, 128, 222, 224, 227, 230, 232, 236, 258, 322, 324, 327, 328, 333, 335, 336, 341, 347, 352, 360, 416, 435, 441, SCMP 118. (Students may count at most one of the following: MATH 105, 106, 108, 116, 128, SCMP 118.) Other courses numbered 200 or above (e.g., special-topics courses) may be counted with the consent of the department.

#### **Statistics**

MATH 106 or an equivalent introductory statistics course; MATH 206; and three courses from the following: MATH 216, 236, 258, 336, 416, 436. (Students may count at most one statistics course from another department. For example, ECON 375 or PSYC 200 may be substituted for one of the courses listed above.)

Our goal is to provide a solid introduction to basic statistical methods, including data analysis, design and analysis of experiments, statistical inference, and statistical models, using professional software such as Minitab, SAS, Maple, and R.

Deviations from the list of approved minor courses must be ratified by the Mathematics Department. Students considering a minor in mathematics or statistics are urged to speak with a member of the department about the selection of courses.

# **Cross-Listed Courses**

The following course is cross-listed in biology and will satisfy the natural science requirement: MATH 258 Mathematical Biology

# Courses

**Note:** This page contains **all** of the regular courses taught by this department. Not all courses are offered every year. Check the <u>searchable schedule</u> to see which courses are being offered in the upcoming semester.

# MATH 105 SURPRISES AT INFINITY

Credit: 0.5 QR

Our intuitions about sets, numbers, shapes, and logic all break down in the realm of the infinite. Seemingly paradoxical facts about infinity are the subject of this course. We will discuss what infinity is, how it has been viewed through history, why some infinities are bigger than others and how a finite shape can have an infinite perimeter. This will very likely be quite different from any mathematics course you have ever taken. Surprises at Infinity focuses on ideas and reasoning rather than algebraic manipulation, though some algebraic work will be required to clarify big ideas. The class will be a mixture of lecture and discussion, based on selected readings. You can expect essay tests, frequent homework, and writing assignments. No prerequisite. Typically offered every other year.

# MATH 106 ELEMENTS OF STATISTICS

Credit: 0.5 QR

This is a basic course in statistics. The topics to be covered are the nature of statistical reasoning, graphical and descriptive statistical methods, design of experiments, sampling methods, probability, probability distributions, sampling distributions, estimation, and statistical inference. Confidence intervals and hypothesis tests for means and proportions will be studied in the one- and two-sample settings. Minitab, a statistical software package, will be used, and students will be engaged in a wide variety of hands-on projects. No prerequisite. Offered every semester.

#### MATH 111 CALCULUS I

Credit: 0.5 QR

The first in a three-semester calculus sequence, this course covers the basic ideas of differential calculus. Differential calculus is concerned primarily with the fundamental problem of determining instantaneous rates of change. In this course we will study instantaneous rates of change from both a qualitative geometric and a quantitative analytic perspective. We will cover in detail the underlying theory, techniques, and applications of the derivative. The problem of anti-differentiation, identifying quantities given their rates of change, will also be introduced. The course will conclude by relating the process of anti-differentiation to the problem of finding the area beneath curves, thus providing an intuitive link between differential calculus and integral calculus. Those who have had a year of high-school calculus but do not have advanced placement credit for MATH 111 should take the calculus placement exam to determine whether they are ready for MATH 112. Students who have .5 unit of credit for calculus may not

receive credit for MATH 111. Prerequisite: solid grounding in algebra, trigonometry, and elementary functions. Students who have credit for MATH 110Y-111Y may not take this course.

#### **MATH 112 CALCULUS II**

Credit: 0.5 QR

The second in a three-semester calculus sequence, this course has two primary foci. The first is integration, including techniques of integration, numerical methods, and applications of integration. This study leads into the analysis of differential equations by separation of variables, Euler's method, and slope fields. The second focus is the notion of convergence, as manifested in improper integrals, sequences, and series, particularly Taylor Series. Prerequisite: MATH 111 or permission of instructor. Offered every semester.

### **MATH 116 STATISTICS IN SPORTS**

Credit: 0.5 QR

Appropriate applications of statistical methods have changed the way some Major League Baseball teams manage the game. (See *Moneyball: The Art of Winning an Unfair Game.*) Statistics are used in other sports to evaluate the performance of individual players or teams. The focus of this course will be on the proper application of statistical models in sports. Students will use appropriate methods to examine interesting questions such as: Are there unusual patterns in the performance statistics of "steroid sluggers" such as Barry Bonds and Mark McGwire or pitchers such as Roger Clemens? Other possible topics include the impact of a penalty kick in soccer, of home field advantage in football, of technological improvements in golf or cycling, and of training methods in marathon running. Although the sport and question of interest will change, the focus on proper applications of appropriate statistical methods will remain the same. Students will analyze data and present their results to the class. Oral and written reports will be expected. No prerequisite. Offered every other year.

Instructor: Hartlaub

# MATH 128 HISTORY OF MATHEMATICS IN THE ISLAMIC WORLD

Credit: 0.5 QR

This course examines an important and interesting part of the history of mathematics and, more generally, the intellectual history of humankind: the history of mathematics in the Islamic world. Some of the most fundamental notions in modern mathematics have their roots here, such as the modern number system, the fields of algebra and trigonometry, and the concept of algorithm, among others. In addition to studying specific contributions of medieval Muslim mathematicians in the areas of arithmetic, algebra, geometry, and trigonometry in some detail, we will examine the context in which Islamic science and mathematics arose, and the role of religion in this development. The rise of Islamic science and its interactions with other cultures (e.g., Greek, Indian, and Renaissance Europe) tell us much about larger issues in the humanities. Thus, this course has both a substantial mathematical component (60-65%) and

a significant history and social science component (35-40%), bringing together three disciplines: mathematics, history, and religion. The course is a part of the Islamic Civilization and Cultures Program, and fulfills the QR requirement. No prerequisite is needed beyond high school algebra and geometry (but a solid knowledge in algebra and geometry is needed).

# **MATH 206 DATA ANALYSIS**

Credit: 0.5 QR

This course focuses on choosing, fitting, assessing, and using statistical models. Simple linear regression, multiple regression, analysis of variance, general linear models, logistic regression, and discrete data analysis will provide the foundation for the course. Classical interference methods that rely on the normality of the error terms will be thoroughly discussed, and general approaches for dealing with data where such conditions are not met will be provided. For example, distribution-free techniques and computer-intensive methods, such as bootstrapping and permutation tests, will be presented. Students will use statistical software throughout the course to write and present statistical reports. The culminating project will be a complete data analysis report for a real problem chosen by the student. The MATH 106-206 sequence provides a thorough foundation for statistical work in economics, psychology, biology, political science, and many other fields. Prerequisite: MATH 106 or MATH 116. Offered every spring.

# **MATH 213 CALCULUS III**

Credit: 0.5 QR

The third in a three-semester calculus sequence, this course examines differentiation and integration in three dimensions. Topics of study include functions of more than one variable, vectors and vector algebra, partial derivatives, optimization, and multiple integrals. Some of the following topics from vector calculus will also be covered as time permits: vector fields, line integrals, flux integrals, curl, and divergence. Prerequisite: MATH 112 or permission of instructor.

#### MATH 216 NONPARAMETRIC STATISTICS

Credit: 0.5 QR

This course will focus on nonparametric and distribution-free statistical procedures. These procedures will rely heavily on counting and ranking techniques. In the one and two sample settings, the sign, signed-rank, and Mann-Whitney-Wilcoxon procedures will be discussed. Correlation and one-way analysis of variance techniques will also be investigated. A variety of special topics will be used to wrap up the course, including bootstrapping, censored data, contingency tables, and the two-way layout. The primary emphasis will be on data analysis and the intuitive nature of nonparametric statistics. Illustrations will be from real data sets, and students will be asked to locate an interesting data set and prepare a report detailing an appropriate nonparametric analysis. Prerequisite: MATH 106, 116, or permission of instructor. Offered every other fall.

#### MATH 218 DATA STRUCTURES AND PROGRAM DESIGN

Credit: 0.5 QR

This course is intended as a second course in programming, as well as an introduction to the concept of computational complexity and the major abstract data structures (such as dynamic arrays, stacks, queues, link lists, graphs, and trees), their implementation and application, and the role they play in the design of efficient algorithms. Students will be required to write a number of programs using a high-level language. Prerequisite: SCMP 118 or permission of instructor. Offered every other spring.

#### **MATH 222 FOUNDATIONS**

Credit: 0.5 OR

This course introduces students to mathematical reasoning and rigor in the context of set-theoretic questions. The course will cover basic logic and set theory, relations--including orderings, functions, and equivalence relations--and the fundamental aspects of cardinality. Emphasis will be placed on helping students in reading, writing, and understanding mathematical reasoning. Students will be actively engaged in creative work in mathematics. Students interested in majoring in mathematics should take this course no later than the spring semester of their sophomore year. Advanced first-year students interested in mathematics are encouraged to consider taking this course in their first year. (Please see a member of the mathematics faculty if you think you might want to do this.) Prerequisite: MATH 213 or permission of instructor. Offered every semester.

# **MATH 224 LINEAR ALGEBRA**

Credit: 0.5 QR

This course will focus on the study of vector spaces and linear functions between vector spaces. Ideas from linear algebra are highly useful in many areas of higher-level mathematics. Moreover, linear algebra has many applications to both the natural and social sciences, with examples arising often in fields such as computer science, physics, chemistry, biology, and economics. In this course, we will use a computer algebra system, such as Maple or Matlab, to investigate important concepts and applications. Topics to be covered include methods for solving linear systems of equations, subspaces, matrices, eigenvalues and eigenvectors, linear transformations, orthogonality, and diagonalization. Applications will be included throughout the course. Prerequisite: MATH 213. Offered every fall.

#### **MATH 227 COMBINATORICS**

Credit: 0.5 OR

Combinatorics is, broadly speaking, the study of finite sets and finite mathematical structures. A great many mathematical topics are included in this description, including graph theory, combinatorial designs, partially ordered sets, networks, lattices and Boolean algebras and combinatorial methods of counting, including combinations and permutations, partitions, generating functions, recurring relations, the

principle of inclusion and exclusion, and the Stirling and Catalan numbers. This course will cover a selection of these topics. Combinatorial mathematics has applications in a wide variety of non-mathematical areas, including computer science (both in algorithms and in hardware design), chemistry, sociology, government, and urban planning; this course may be especially appropriate for students interested in the mathematics related to one of these fields. Prerequisite: MATH 112 or permission of instructor. Offered every other spring.

# MATH 230 EUCLIDEAN AND NON-EUCLIDEAN GEOMETRY

Credit: 0.5 QR

The *Elements* of Euclid, written over two thousand years ago, is a stunning achievement. The *Elements* and the non-Euclidean geometries discovered by Bolyai and Lobachevsky in the nineteenth century form the basis of modern geometry. From this start, our view of what constitutes geometry has grown considerably. This is due in part to many new theorems that have been proved in Euclidean and non-Euclidean geometry but also to the many ways in which geometry and other branches of mathematics have come to influence one another over time. Geometric ideas have widespread use in analysis, linear algebra, differential equations, topology, graph theory, and computer science, to name just a few areas. These fields, in turn, affect the way that geometers think about their subject. Students in MATH 230 will consider Euclidean geometry from an advanced standpoint, but will also have the opportunity to learn about non-Euclidean geometries. Prerequisite: MATH 222 or permission of instructor. For possible offering times, please consult the department chair.

# MATH 231 MATHEMATICAL PROBLEM SOLVING

Credit: 0.25

Looking at a problem in a creative way and seeking out different methods toward solving it are essential skills in mathematics and elsewhere. In this course, students will build their problem-solving intuition and skills by working on challenging and fun mathematical problems. Common problem-solving techniques in mathematics will be covered in each class meeting, followed by collaboration and group discussions, which will be the central part of the course. The course will culminate with the Putnam exam on the first Saturday in December. Interested students who have a conflict with that date should contact the instructor. Prerequisite: MATH 112 or equivalent.

#### **MATH 232 VECTOR CALCULUS**

Credit: 0.5 QR

Physical and natural phenomena depend on a complex array of factors, and to analyze these factors requires the understanding of geometry in two and three (or more) dimensions. This course will continue the study of multivariable calculus begun in MATH 213. Topics of study will include vector fields, line and surface integrals, potential functions, classical vector analysis, and Fourier Series. Computer labs will

be incorporated throughout the course, and physical applications will be plentiful. Prerequisite: MATH 213. For possible offering times, please consult the department chair.

# **MATH 236 RANDOM STRUCTURES**

Credit: 0.5 QR

This course will explore the theory, structure, applications, and interesting consequences when probability is introduced to mathematical objects. Some of the core topics will be random graphs, random walks and Markov processes, as well as randomness applied to sets, permutations, polynomials, functions, integer partitions, and codes. Previous study of all of these mathematical objects is not a prerequisite, as essential background will be covered during the course. In addition to studying the random structures themselves, a concurrent focus of the course will be the development of mathematical tools to analyze them, such as combinatorial concepts, indicator variables, generating functions, discrete distributions, laws of large numbers, asymptotic theory, and computer simulation. Prerequisite: MATH 112 or permission of instructor. Offered every other year.

# **MATH 258 MATHEMATICAL BIOLOGY**

Credit: 0.5 QR

In biological sciences, mathematical models are becoming increasingly important as tools for turning biological assumptions into quantitative predictions. In this course, students will learn how to fashion and use these tools to explore questions ranging across the biological sciences. We will survey a variety of dynamic modeling techniques, including both discrete and continuous approaches. Biological applications may include population dynamics, molecular evolution, ecosystem stability, epidemic spread, nerve impulses, sex allocation, and cellular transport processes. The course is appropriate both for math majors interested in biological applications, and for biology majors who want the mathematical tools necessary to address complex, contemporary questions. As science is becoming an increasingly collaborative effort, biology and math majors will be encouraged to work together on many aspects of the course. Coursework will include homework problem-solving exercises and short computational projects. Final independent projects will require the development and extension of an existing biological model selected from the primary literature, using mathematical software like Mathematica, Matlab, R, or Maple. Students will make a poster presentation of their results. Prerequisite: This course will build on (but not be limited by) an introductory-level knowledge base in both subjects, including MATH 111 and either BIOL 112 or BIOL 113. Interested biology and math majors lacking one of the prerequisites are encouraged to consult with the instructor. Offered every other year.

# **MATH 322 MATHEMATICAL LOGIC**

Credit: 0.5

This course is a mathematical examination of the formal language most common in mathematics: predicate calculus. We will examine various definitions of meaning and proof for this language, and

consider its strengths and inadequacies. We will develop some elementary computability theory en route to rigorous proofs of Godel's Incompleteness Theorems. Prerequisite: MATH 222 or PHIL 120 or permission of instructor. Offered occasionally.

# **MATH 324 LINEAR ALGEBRA II**

Credit: 0.5 QR

This course builds on the concepts that arise in MATH 224. Topics will vary and will likely include some of the following: abstract vector spaces, inner product spaces, linear mappings and canonical forms, linear models, linear codes, the singular value decomposition, wavelets. Prerequisite: MATH 224. Offered every other year.

#### **MATH 327 NUMBER THEORY SEMINAR**

Credit: 0.5 QR

Patterns within the set of natural numbers have enticed mathematicians for well over two millennia, making number theory one of the oldest branches of mathematics. Rich with problems that are easy to state but fiendishly difficult to solve, the subject continues to fascinate professionals and amateurs alike. In this course, we will get a glimpse at both the old and the new. In the first two-thirds of the semester, we will study topics from classical number theory, focusing primarily on divisibility, congruences, arithmetic functions, sums of squares, and the distribution of primes. In the final weeks we will explore some of the current questions and applications of number theory. We will study the famous RSA cryptosystem, and students will be reading and presenting some current (carefully chosen) research papers. Prerequisite: MATH 222. Offered every other year.

# MATH 328 AN INTRODUCTION TO CODING THEORY AND CRYPTOGRAPHY

Credit: 0.5 QR

Coding theory, or the theory of error-correcting codes, and cryptography are two recent applications of algebra and discrete mathematics to information and communications systems. The goals of this course are to introduce students to these subjects and to understand some of the basic mathematical tools used. While coding theory is concerned with the reliability of communication, the main problem of cryptography is the security and privacy of communication. Applications of coding theory range from enabling the clear transmission of pictures from distant planets to quality of sound in compact disks. Cryptography is a key technology in electronic security systems. Topics likely to be covered include basics of block coding, encoding and decoding, linear codes, perfect codes, cyclic codes, BCH and Reed-Solomon codes, and classical and public-key cryptography. Other topics may be included depending on the availability of time and the background and interests of the students. Other than some basic linear algebra, the necessary mathematical background (mostly abstract algebra) will be covered within the course. Prerequisite: MATH 224 or permission of instructor. Offered every other year.

# **MATH 333 DIFFERENTIAL EQUATIONS**

Credit: 0.5 QR

Differential equations arise naturally to model dynamical systems such as often occur in physics, biology, chemistry, and economics, and have given major impetus to other fields in mathematics, such as topology and the theory of chaos. This course covers basic analytic, numerical, and qualitative methods for the solution and understanding of ordinary differential equations. Computer-based technology will be used. Prerequisite: MATH 224 or PHYS 245 or permission of instructor. Offered every other spring.

#### MATH 335 ABSTRACT ALGEBRA I

Credit: 0.5 OR

Abstract algebra is the study of algebraic structures that describe common properties and patterns exhibited by seemingly disparate mathematical objects. The phrase "abstract algebra" refers to the fact that some of these structures are generalizations of the material from high school algebra relating to algebraic equations and their methods of solution. In Abstract Algebra I, we focus entirely on group theory. A group is an algebraic structure that allows one to describe symmetry in a rigorous way. The theory has many applications in physics and chemistry. Since mathematical objects exhibit pattern and symmetry as well, group theory is an essential tool for the mathematician. Furthermore, group theory is the starting point in defining many other more elaborate algebraic structures including rings, fields, and vector spaces. In this course, we will cover the basics of groups, including the classification of finitely generated abelian groups, factor groups, the three isomorphism theorems, and group actions. The course culminates in a study of Sylow theory. Throughout the semester there will be an emphasis on examples, many of them coming from calculus, linear algebra, discrete math, and elementary number theory. There will also be a couple of projects illustrating how a formal algebraic structure can empower one to tackle seemingly difficult questions about concrete objects (e.g., the Rubik's cube or the card game SET). Finally, there will be a heavy emphasis on the reading and writing of mathematical proofs. Prerequisite: MATH 222 or permission of instructor. Junior standing is recommended. Offered every other fall.

# **MATH 336 PROBABILITY**

Credit: 0.5 QR

This course provides a calculus-based introduction to probability. Topics include basic probability theory, random variables, discrete and continuous distributions, mathematical expectation, functions of random variables, and asymptotic theory. Prerequisite: MATH 213. Offered every fall.

# **MATH 341 REAL ANALYSIS I**

Credit: 0.5 QR

This course is a first introduction to real analysis. "Real" refers to the real numbers. Much of our work will revolve around the real number system. We will start by carefully considering the axioms that describe it.

"Analysis" is the branch of mathematics that deals with limiting processes. Thus the concept of distance will also be a major theme of the course. In the context of a general metric space (a space in which we can measure distances), we will consider open and closed sets, limits of sequences, limits of functions, continuity, completeness, compactness, and connectedness. Other topics may be included, if time permits. Prerequisite: MATH 213 and 222. Junior standing is recommended. Offered every year.

# **MATH 347 MATHEMATICAL MODELS**

Credit: 0.5 QR

This course introduces students to the concepts, techniques, and power of mathematical modeling. Both deterministic and probabilistic models will be explored, with examples taken from the social, physical, and life sciences. Students engage cooperatively and individually in the formulation of mathematical models and in learning mathematical techniques used to investigate those models. Prerequisite: MATH 106 and MATH 224 or MATH 258 or permission of instructor. Offered every other year.

# **MATH 352 COMPLEX FUNCTIONS**

Credit: 0.5 QR

The course starts with an introduction to the complex numbers and the complex plane. Next students are asked to consider what it might mean to say that a complex function is differentiable (or analytic, as it is called in this context). For a complex function that takes a complex number z to f(z), it is easy to write down (and make sense of) the statement that f is analytic at z if

exists. Subsequently, we will study the amazing results that come from making such a seemingly innocent assumption. Differentiability for functions of one complex variable turns out to be a very different thing from differentiability in functions of one real variable. Topics covered will include analyticity and the Cauchy-Riemann equations, complex integration, Cauchy's theorem and its consequences, connections to power series, and the residue theorem and its applications. Prerequisite: MATH 224. Offered every other year.

# **MATH 360 TOPOLOGY**

Credit: 0.5 QR

Topology is a relatively new branch of geometry that studies very general properties of geometric objects, how these objects can be modified, and the relations between them. Three key concepts in topology are compactness, connectedness, and continuity, and the mathematics associated with these concepts is the focus of the course. Compactness is a general idea helping us to more fully understand the concept of limit, whether of numbers, functions, or even geometric objects. For example, the fact that a closed interval (or square, or cube, or n-dimensional ball) is compact is required for basic theorems of calculus. Connectedness is a concept generalizing the intuitive idea that an object is in one piece: the most famous of all the fractals, the Mandelbrot Set, is connected, even though its best computer-graphics representation might make this seem doubtful. Continuous functions are studied in calculus, and the

general concept can be thought of as a way by which functions permit us to compare properties of different spaces or as a way of modifying one space so that it has the shape or properties of another. Economics, chemistry, and physics are among the subjects that find topology useful. The course will touch on selected topics that are used in applications. Prerequisite: MATH 222 or permission of instructor. Offered every two to three years, depending on student interest.

# **MATH 416 LINEAR REGRESSION MODELS**

Credit: 0.5 QR

This course will focus on linear regression models. Simple linear regression with one predictor variable will serve as the starting point. Models, inferences, diagnostics, and remedial measures for dealing with invalid assumptions will be examined. The matrix approach to simple linear regression will be presented and used to develop more general multiple regression models. Building and evaluating models for real data will be the ultimate goal of the course. Time series models, nonlinear regression models, and logistic regression models may also be studied if time permits. Prerequisite: MATH 106, 213, and 224 or permission of instructor. Offered every other spring.

# **MATH 435 ABSTRACT ALGEBRA II**

Credit: 0.5 QR

Abstract Algebra II picks up where MATH 335 ends, focusing primarily on rings and fields. Serving as a good generalization of the structure and properties exhibited by the integers, a ring is an algebraic structure consisting of a set together with two operations--addition and multiplication. If a ring has the additional property that division is well-defined, one gets a field. Fields provide a useful generalization of many familiar number systems: the rational numbers, the real numbers, and the complex numbers. Topics to be covered include: polynomial rings; ideals; homomorphisms and ring quotients; Euclidean domains, principal ideal domains, unique factorization domains; the Gaussian integers; factorization techniques and irreducibility criteria. The final block of the semester will serve as an introduction to field theory, covering algebraic field extensions, symbolic adjunction of roots; construction with ruler and compass; and finite fields. Throughout the semester there will be an emphasis on examples, many of them coming from calculus, linear algebra, discrete math, and elementary number theory. There will also be a heavy emphasis on the reading and writing of mathematical proofs. Prerequisite: MATH 335. Offered every other spring.

# **MATH 436 MATHEMATICAL STATISTICS**

Credit: 0.5 OR

This course follows MATH 336 and introduces the mathematical theory of statistics. Topics include sampling distributions, order statistics, point estimation, maximum likelihood estimation, methods for comparing estimators, interval estimation, moment generating functions, bivariate transformations, likelihood ratio tests, and hypothesis testing. Computer simulations will accompany and corroborate

many of the theoretical results. Course methods will often be applied to real data sets. Prerequisite: MATH 336. Offered every other spring.

#### **MATH 441 REAL ANALYSIS II**

Credit: 0.5 QR

This course follows Real Analysis I. Topics will include a study differentiation and (Riemann) integration of functions of one variable, sequences and series of functions, power series and their properties, iteration and fixed points. Other topics may be included as time permits. For example: a discussion of Newton's method or other numerical techniques; differentiation and integration of functions of several variables; spaces of continuous functions; the implicit function theorem; and everywhere continuous, nowhere differentiable functions. Prerequisite: MATH 341. Offered every other spring.

### **MATH 493 INDIVIDUAL STUDY**

Credit: 0.25-0.5

Individual study is a privilege reserved for students who want to pursue a course of reading or complete a research project on a topic not regularly offered in the curriculum. It is intended to supplement, not take the place of, coursework. Individual study cannot normally be used to fulfill requirements for the major. Typically, individual study will earn .5 unit or .25 unit of credit.

To qualify, a student must identify a member of the Mathematics Department willing to direct the project. The professor, in consultation with the student, will create a tentative syllabus (including a list of readings and/or problems, goals, and tasks) and describe in some detail the methods of assessment (e.g. problem sets to be submitted for evaluation biweekly; a twenty-page research paper submitted at the course's end, with rough drafts due at given intervals, etc.). Individual studies also require the approval of the department chair. The department expects the student to meet regularly with his or her instructor for at least one hour per week, or the equivalent.

Students must begin discussion of their proposed individual study well in advance, preferably the semester before the course is to take place. Prerequisite: permission of instructor and department chair.

# **MATH 498 SENIOR HONORS**

Credit: 0.25 QR

This course will consist largely of an independent project in which students read several sources to learn about a mathematical topic that complements material studied in other courses, usually an already completed depth sequence. This study will culminate in an expository paper and a public or semi-public presentation before an audience consisting of at least several members of the mathematics faculty as well as an outside examiner. Prerequisite: At least one "depth sequence" completed and permission of the department.